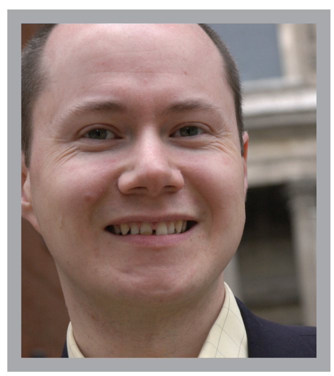
In the first of a two-part series on hedging risk, *Neil Palmer* looks at the effects of imperfect correlation on basis risk, and finds that unless you have a perfect hedge, you may just have to learn to live with risk

Slaying the dragon

★ The Nobel prize-winner Niels Bohr supposedly said: "Prediction is very difficult, especially if it's about the future." If Professor Bohr had been an energy quant instead of a quantum physicist, he might have said: "Valuation is very difficult, especially if it involves risk."

Risk is a tremendous pain in the neck for quants: it's not easy to put a price on uncertainty. On the other hand, if it wasn't for risk, then we poor quants would be out of a job.

The quant's approach is to try to chase risk away – to hedge it somehow, keep track of how much is spent in the process and then put a price tag on the risk that's left. In this two-part series of Quant Talk we'll look at two general cases of hedging risk. Here, we confine ourselves to linear positions and look at the size of the basis risk that arises from any less-than-perfect correlation.



Everyone knows that a great way to cut out risk is by hedging with a positively correlated asset. Suppose you're short a forward obligation in something that's not too liquid.

Maybe it's a particular type of physical coal or an unusual electricity load shape. Your idea is to hedge it by going long a standard financial coal contract – such as API#2 – or baseload power.

So far so good – there's probably a decent-looking correlation between your exposure and the value of your hedge. But is it decent enough?

Let's say you can estimate your original risk – that is, the risk you're running with an unhedged position. And suppose also that you know the correlation between the value of your illiquid exposure and the price of a hedging asset.

If you want to minimise your risk, then the size of your hedge position should depend on the level of correlation itself. This is because the hedge, if imperfectly correlated, introduces some new risk all of its own. As correlation rises, you should buy more of the hedging asset. And, barring accidents, the total amount of risk on your book should fall. But this total risk falls more like a feather than a stone.

It might be surprising just how high correlation has to be in order to slay the dragon of risk. In February's Quant Talk we said that it can sometimes be more helpful to think of the square of the correlation coefficient ρ as a 'gut-feel' indicator of how powerful a certain market price relationship is. Since we're talking about positive but imperfect correlation, it's plain that ρ^2 is smaller than ρ itself.

This emphasises the point that the real impact of correlation is often less than that which the numerical value of the coefficient might suggest. It's a good rule of thumb for moderate values of correlation, and it provides an upper boundary on the amount of risk that correlation can 'kill'. For example, if correlation is only 10% and you put on your best hedge, you can only expect to reduce overall risk by less than 1%.

But our rule-of-thumb can appear over-optimistic when correlation is larger. If correlation is 50%, then the square is 25% – but it turns out that the best hedge kills only about 13% of the original risk. And if correlation is 80% then our best hedge manages to kill only 40% of the risk, leaving us exposed to the remaining 60%. Even with a correlation of 95%, our best hedge still leaves us with more than 30% of the original risk. It's clear that it becomes very difficult or exorbitantly expensive to remove that very last drop of risk. Table 1 and figure 1 show this.[1]

Here's another rule of thumb. This one works in every case. Did you hear about the claim that option time-value reduces by about a third whenever you halve the time-to-expiry? Well, this one's in the same vein.

Consider again the square of the correlation coefficient – and suppose the square moves 'halfway to 100%'. For example, starting with 50% correlation, we square and get 25%, and now choose the mid-point between 25% and 100%. This is 62.5%. Now take the square root, and we get about 79%. So we've moved from 50% to 79% correlation. Similarly, if we'd started with 80% correlation, this procedure would have moved us to about 91% correlation.

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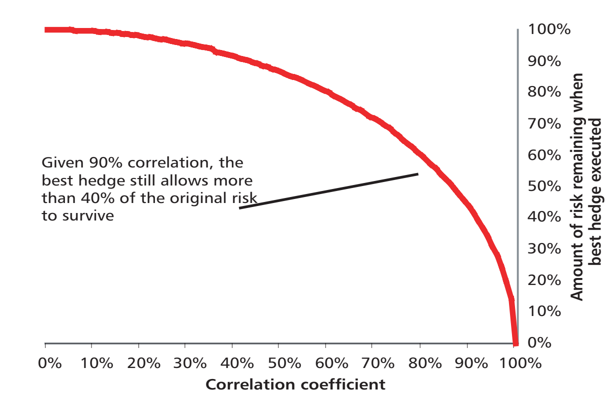
Neil Palmer

> **"The cost involved in reducing uncertainty by searching for the best possible hedge may exceed the benefits of reducing the risk. In other words, we may just have to learn to live with some risk"**
>
> Neil Palmer

If you want to kill *this* much of the original risk ...	... you need a hedge with *this* much correlation
25%	66%
50%	87%
75%	97%
80%	98%
90%	99.5%
95%	99.9%
99%	99.99%

T1. Hedge correlation



F1. How much basis risk does correlation kill?

And here's the rule: in such a case, whatever your starting point, the amount of residual risk drops by just under a third when the square of correlation moves halfway to 100%. Moving from 50% to 79% correlation reduces the best-hedge residual risk from about 87% to 61% – a reduction of just under a third.

There's an important 'postscript' to all these remarks. So far, we've been quietly assuming that the volatility of both your underlying position and that of the hedge asset is known clearly, and is not subject to uncertainty. Yet the fluctuation of volatility is itself another risk, to say nothing of the fluctuation of correlation. This means that from a practical point of view, all our estimates of residual risk will still be on the low side. Oh, and we haven't even mentioned volume or credit risks. Not surprisingly, dragon-slaying is tougher in practice than in theory.

There is a saving grace, however. Sometimes, because of the underlying physical relationship between energy commodities, the fundamental correlation between energy exposures can be greater than we often estimate, especially over long time periods. If we use pairs of daily asset price changes to estimate correlation, then it is possible we may understate the real correlation. So our hedges may not perform quite so badly as the discussion here might suggest.

Learning to live with risk

Yet the message is clear: unless you really do have a perfect hedge, the quest for a risk-free life is like the search for the Holy Grail. Don't devote too much of your annual leave to it. If quantum physics has its own Uncertainty Principle, then surely our energy world has a right to one, too: the cost and effort involved in reducing uncertainty to a very small level by searching high and low for the best possible hedge may exceed the benefits of reducing that risk in the first place. In other words, we may just have to learn to live with some risk.

Next time in Quant Talk we take a look at how not even perfect correlation can save you if your exposure is non-linear, such as when you're hedging an option liability.

Ever since 1973 there's been a lot of hoo-ha about a wonderful option pricing formula and an apparently marvellous cure-all called dynamic hedging. Is this the answer to our prayers – the magical elixir of life? ER

1. How have we calculated these numbers? Say your original risk – that is, the liability to which you're exposed – is given by a random variable with variance 1. Call this X. Then suppose you have another random variable available, Y, which also has variance 1 and a positive correlation of ρ with X. You can show that the linear combination of these variables with the least total variance is $X-\rho \times Y$. This means that the best hedge ratio is given by the correlation coefficient itself. And in this case, the amount of residual risk (expressed as a standard deviation) is given by $\sqrt{1-\rho^2}$. This is the function used to generate the graph and table in this article.

2. This result follows directly from the analysis in footnote 1. Actually, the precise amount of risk reduction is $\left(1 - 1/\sqrt{2}\right)\times 100\% = 29.3\%$, but somehow when we're talking about risk it feels wrong to aim for too much precision.

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