In this first article of the new gas storage segment of the Masterclass series, *John Breslin, Les Clewlow, Tobias Elbert, Calvin Kwok* and *Chris Strickland* provide an illustration of how the four most common valuation methodologies can be used to optimise gas storage and trading

Gas storage: overview and static valuation

★ Gas storage serves several purposes in the gas industry. Traditionally, storage facilities are used to move production capacity from one point in time to another, such as to shift the supply to the demand peaks in winter periods. They also provide a buffer against unexpected changes in demand or supply, for example, by providing distribution companies with extra supply during periods of heavy demand by supplementing pipeline capacity. The unexpected changes in demand may, for example, be due to unseasonal weather or industrial users with large short-term swings in gas requirements. Unexpected changes in supply can occur due to accidents to plant and equipment, or disruption of production caused by natural disasters.

Over the past 10 to 15 years, deregulation of gas markets has meant that storage facilities are now available for commercial use in addition to operational use, and so gas storage now has an additional purpose in that it allows traders to exploit predictable seasonal variations in the market price of gas. This in turn leads to the need to value storage facilities. In this and the two following articles we will provide an illustration of how the four most common valuation methodologies are used in practice, with practical examples illustrating their implementation.

Storage constraints

One of the keys to accurately valuing storage facilities is to correctly incorporate the constraints. Typical storage constraints include:

• **Capacity** – this is the total amount of working gas that can be utilised in the facility.

• **Injection and withdrawal rates** – these determine the speed at which gas can be injected or withdrawn from the storage facility. In general, the rates are not constant, but can differ by the time of year or, more usually, by the amount of gas that is in storage (generally referred to as 'ratchets') – as we fill the storage facility, the rate at which we can make further injections falls, while the rate at which we can withdraw gas increases. These rates can also differ markedly between different types of storage. For example, aquifiers, or depleted fields, are amongst the slowest of the different types of storage facilities, while salt domes are amongst the fastest, allowing multi-cycles and a fast response to changes in cash and forward prices.

In addition to the constraints, the model also needs to account for the costs of injection and withdrawal. These can be both fixed costs reflecting the operating costs of the facility, and variable costs that reflect transport costs or the cost for the energy required to pump gas in or out of the storage.

Modelling considerations

We can think of the valuation of storage as being split into two components. The first is to model the evolution of the underlying gas prices. Preferably this should model the evolution of the underlying gas spot price and prices for forward contracts in a way that is consistent with our other modelling assumptions. This ensures that in a portfolio context we can consistently value the storage facility along with exchange-traded options, gas daily options, swing contracts, etc, and also incorporate these instruments into the risk metrics of value-at-risk or earnings-at-risk. In this way we avoid the inconsistency of different models for different products and also a disconnect between valuation and risk management reporting.

Once a model for the gas price has been determined, the second stage of the valuation is the technique or techniques that we use to capture the constraints of the storage and derive the trading strategy. In simple terms, the trading strategy is to optimise the value of buying gas at low prices and injecting it into the facility and withdrawing gas and selling at high prices, subject to the volume constraints of the facility and the injection and withdrawal constraints. A further complicating factor is that often not all of the gas can be used for capturing market opportunities – some of the gas might be needed to fulfil reliability requirements.

Overview of methodologies

Given these requirements, the four common valuation methodologies that we will discuss in this and subsequent articles are:

Intrinsic valuation

Sometimes called forward optimisation, the intrinsic valuation methodology is intuitive and simple to understand and derives its value from seasonal or time spreads in the price of gas. Months for which the forward price for gas is relatively low are chosen from the current forward curve to enter into long positions in order to buy gas and inject into the facility. These are in turn sold forward to the months for which the forward price is relatively high, when the gas is withdrawn from storage. Note that we can use bid and offer curves to properly account for the buy and sell prices at which we can trade gas. The intrinsic value is known and fixed on the first day, but it ignores the inherent flexibility yielded by the facility in changing market conditions and hence does not capture value that could be obtained from these changes.

Basket of spread options

Analogous to the intrinsic value that optimises the position in the forward contracts, in this strategy we derive the optimal portfolio of calendar spread options, subject to the storage constraints. Storage then is represented as a long position[1] in a basket of calendar spread options, and in practice these spread options are delta hedged to capture the expected value of the option position.

Rolling intrinsic and rolling basket of spreads

The rolling intrinsic strategy is an extension to the intrinsic strategy that recognises the changing value in the intrinsic spreads as the forward curve evolves. Under this strategy the user recognises any value increases in the spreads of different months and the mark-to-market cashflows, by closing out existing positions and entering new positions to lock in the new (higher) overall value. The rolling basket of spreads strategy is developed similarly. A Monte Carlo simulation of forward prices is used to build up a distribution of values, enabling both an expected value as well as a distribution of values to be obtained. Although these rolling strategies can capture extra value as the market prices evolve, we note that they are suboptimal, since each rebalancing takes no account of potential future trades.

Spot optimisation

While the previous strategies rely on taking positions in the forward market, in this approach we model the value that can be obtained from making daily decisions of the injection and withdrawal of spot gas. This approach aims to optimise those spot trading decisions to maximise the total discounted revenue over the life of the storage contract, across all possible price paths.[2] By using an underlying spot price model that is consistent with, and calibrated to the market forward curve, we ensure that the value obtained is consistent with the forward strategies described above. In particular, if we consider the case of zero volatility in the spot price this strategy is equivalent to the intrinsic valuation approach.

Typically there are two main approaches to implementing solutions for the optimal spot strategy. The first is by using backwards induction in conjunction with trinomial trees, and the second is by employing least squares regression in a Monte Carlo simulation framework.

In the remainder of this article we will illustrate the first two strategies above with a practical example, and the pros and cons of the last two strategies will be discussed in subsequent articles in our Masterclass series.

To illustrate the intrinsic valuation methodology we consider a storage facility with the following characteristics and constraints:

- Total capacity: 1,000,000 MMBtu (or 1 Bcf)[3]
- Maximum injection rate: 8,197 MMBtu/day (i.e. 122 days to fill the facility)
- Maximum withdrawal rate: 16,393 MMBtu/day (i.e. 61 days to empty the facility)
- Injection cost: 0.010 pence/therm
- Withdrawal cost: 0.006 pence/therm
- The valuation period is from April 1, 2007 to March 31, 2008, with the valuation being performed as at March 31, 2007.
- The original and terminal constraints are that the facility must be empty on the start and end dates.
- Assume a flat discount rate of 3.5% for the valuation period.

Note that for clarity and ease of explanation, we have not included ratchets on the injection and withdrawal rates.

The intrinsic strategy is to optimise a hedge on the forward markets for the valuation date, and the resulting value is the intrinsic value that could be realised if sold forward today. To describe the facility in the optimisation we use the following notations:

V	Storage facility capacity
$I_{\max}$	Maximum daily injection rate
$W_{\max}$	Maximum daily withdrawal rate
c_I	Cost of injection
c_W	Cost of withdrawal

In order to set up the optimisation problem we also define the following:

ΔF_{ij}	Discounted spreads for injection in month i and withdrawal in month j
v_{ij}	Position in spread
I_i	Total injection at month i
W_j	Total withdrawal at month j
V_i	Storage level at month i

1. A long position in the calendar-spread option is defined as being long the near-dated contract and short the far-dated contract.

2. Note that the spot strategy can be converted to an equivalent forward strategy by delta hedging in the forward market.

3. MMBtu stands for millions of British thermal units and Bcf stands for billion cubic feet.

The optimisation problem then becomes to maximise the cashflow, which is achieved via:

$$\max_{v_{ij}} \sum_i \sum_j v_{ij} \, \Delta F_{ij}$$

subject to the following constraints:

$$v_{ij} \geq 0$$
$$I_i = \sum_j v_{ij} \leq I_{\max}$$
$$W_j = \sum_i v_{ij} \leq W_{\max}$$
$$V_i \leq V$$

That is, we want to maximise the following factors: the cashflows accruing to the operation of the facility subject to the constraints that all the positions taken are positive; that the injection positions summed across all months are less than the maximum monthly injection; the withdrawal positions summed across all months are less than the maximum monthly withdrawal; and that the level of storage in any month does not exceed the capacity. Additionally, for this problem we need to add constraints to ensure the facility is empty at the beginning and end of the contract.

Figure 1 shows the forward curves we will use for this example and represents the UK's NBP gas curve from March 31, 2007. Note that we can use separate bid and ask curves if it is important to account for the bid-ask spread when calculating the seasonal spread for the trades.[4] The forward curve shows a typical shape for gas forward prices, that is, low prices during summer followed by high prices during winter. Qualitatively it is easy to determine what the intrinsic strategy should be for this example: take a long position for injection during the summer months, and take a short position during the winter months to withdraw the stored gas. However, in order to determine the precise strategy and the value of that strategy it is necessary to solve the optimisation problem defined above.

As the storage example used here does not involve ratchets, and in order to show the detailed calculations, we set this example up on a spreadsheet and solve for the optimal forward positions using the Solver in Excel. For the example above, we constructed a table of the discounted monthly spreads, which we display

F1. Bid and Ask forward quotes for the intrinsic valuation *Source: Lacima Group*

4. Typical bid-ask spreads are only 1 – 2% of the price, so they have minimal impact on the valuations in our examples, however the spreads can become larger if liquidity in the market is reduced.

T1. Discounted forward spreads associated with all unordered pairs of forward prices (pence/therm) *Source: Lacima Group*

Injection		May 07	Jun 07	Jul 07	Aug 07	Sep 07	Oct 07	Nov 07	Dec 07	Jan 08	Feb 08	Mar 08
	Apr 07	–0.12	0.17	–0.91	–0.55	–2.30	7.22	23.52	29.22	32.29	30.47	23.80
	May 07	-	0.13	–0.94	–0.59	–2.34	7.19	23.49	29.18	32.25	30.43	23.76
	Jun 07	-	-	–1.20	–0.84	–2.59	6.93	23.23	28.92	31.99	30.17	23.50
	Jul 07	-	-	-	–0.11	–1.86	7.66	23.96	29.66	32.73	30.91	24.24
	Aug 07	-	-	-	-	–2.39	7.14	23.43	29.13	32.20	30.38	23.71
	Sep 07	-	-	-	-	-	8.90	25.20	30.89	33.96	32.14	25.47
	Oct 07	-	-	-	-	-	-	15.38	21.07	24.14	22.32	15.65
	Nov 07	-	-	-	-	-	-	-	4.59	7.66	5.84	–0.83
	Dec 07	-	-	-	-	-	-	-	-	1.54	-0.28	–6.95
	Jan 08	-	-	-	-	-	-	-	-	-	-3.54	–10.21
	Feb 08	-	-	-	-	-	-	-	-	-	-	–7.88

in table 1. Each value shown in the table represents the discounted revenue the owner of the facility would receive by injecting one unit of gas and withdrawing it at a later time. For instance, the last value in the first row corresponds to injection in Apr 07 and withdrawal in Mar 08 and is given by:

$$\begin{aligned}\Delta F_{Apr07,\,Mar08} &= DF_{Mar08}\left(F^{Bid}_{Mar08} - c_W\right) \\ &- DF_{Apr07}\left(F^{Ask}_{Apr07} + c_I\right)\end{aligned}$$

where F^{Bid}_{Mar08} and F^{Ask}_{Apr07} respectively represent the bid price and ask price for Mar 08 and Apr 07, and DF_{Mar08} and DF_{Apr07} are the associated discount factors.

Having created a table of the discounted forward spreads, the next step is to find the set of optimal volumes to lock in for each of the calendar spreads. These optimal values are obtained by using the Excel Solver to maximise the sum of the total revenues subject to the injection and withdrawal constraints and additional capacity constraint not shown in the tables. Table 2 shows the resulting volume set and the monthly injection and withdrawal constraints. The corresponding set of optimised discounted revenues that the storage owner will receive is given in table 3. If we sum the individual revenues we find the total value of the facility using the intrinsic strategy is £3,190,696.

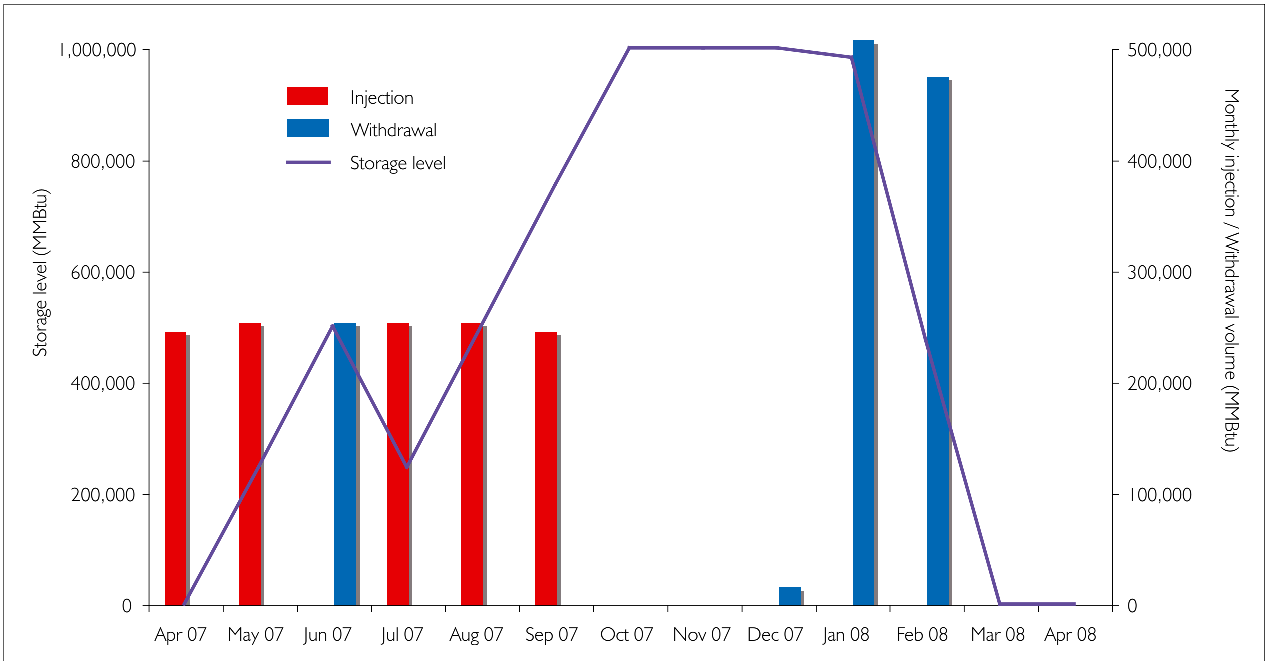
As expected, the solution generally requires injection during the low priced summer months and withdrawal during the high priced winter months. The injection and withdrawal volumes along with the movement of the storage level are plotted in figure 2. Note that the model also

T2. Optimal monthly injections and withdrawals volume under the intrinsic strategy (MMBtu) *Source: Lacima Group*

		May 07	Jun 07	Jul 07	Aug 07	Sep 07	Oct 07	Nov 07	Dec 07	Jan 08	Feb 08	Mar 08	ΣInjection	Constraint
	Apr 07	–	245,910	-	-	-	-	-	-	-	-	-	245,910	245,910
	May 07	-	8,231	-	-	-	-	-	16,393	229,483	-	-	254,107	254,107
	Jun 07	-	-	-	-	-	-	-	-	-	-	-	-	245,910
	Jul 07	-	-	-	-	-	-	-	-	-	254,107	-	254,107	254,107
	Aug 07	-	-	-	-	-	-	-	-	254,107	-	-	254,107	254,107
Injection	Sep 07	-	-	-	-	-	-	-	-	24,607	221,303	-	245,910	245,910
	Oct 07	-	-	-	-	-	-	-	-	-	-	-	-	254,107
	Nov 07	-	-	-	-	-	-	-	-	-	-	-	-	245,910
	Dec 07	-	-	-	-	-	-	-	-	-	-	-	-	254,107
	Jan 08	-	-	-	-	-	-	-	-	-	-	-	-	254,107
	Feb 08	-	-	-	-	-	-	-	-	-	-	-	-	237,713
	ΣWithdrawal	-	254,141	-	-	-	-	-	16,393	508,197	475,410	-		
	Constraint	491,803	508,197	491,803	508,197	508,197	491,803	508,197	491,803	508,197	508,197	475,410		

T3. Position adjusted revenues under the intrinsic strategy (£) *Source: Lacima Group*

		Withdrawal											
		May 07	Jun 07	Jul 07	Aug 07	Sep 07	Oct 07	Nov 07	Dec 07	Jan 08	Feb 08	Mar 08	ΣInjection
	Apr 07	-	4,154	-	-	-	-	-	-	-	-	-	4,154
	May 07	-	109	-	-	-	-	-	47,836	740,092	-	-	788,038
	Jun 07	-	-	-	-	-	-	-	-	-	-	-	-
	Jul 07	-	-	-	-	-	-	-	-	-	785,372	-	785,372
	Aug 07	-	-	-	-	-	-	-	-	818,193	-	-	818,193
Injection	Sep-07	-	-	-	-	-	-	-	-	83,575	711,364	-	794,939
	Oct 07	-	-	-	-	-	-	-	-	-	-	-	-
	Nov 07	-	-	-	-	-	-	-	-	-	-	-	-
	Dec 07	-	-	-	-	-	-	-	-	-	-	-	-
	Jan 08	-	-	-	-	-	-	-	-	-	-	-	-
	Feb 08	-	-	-	-	-	-	-	-	-	-	-	-
	ΣWithdrawal	-	4,264	-	-	-	-	-	47,836	1,641,860	1,496,736	-	



F2. Optimised injection and withdrawal strategy and resulting storage level *Source: Lacima Group*

produces a small amount of positive cashflow via some withdrawal in Jun 07, which may not have been obvious without carrying out the optimisation.

The intrinsic value methodology is a 'set and forget' strategy that ignores the flexibility in the storage facility. One way of capturing the extra – sometimes called real option, or extrinsic – value that can be attributed to this flexibility is by the basket of spread option strategy. In this strategy the value of storage is derived as the expected payoff to an optimally derived portfolio of calendar spread options[5] rather than their underlying forward spreads. We thus allocate our injection and withdrawal decisions so that we obtain the optimal combination of spread options. The resulting portfolio value is perfectly hedged by the underlying storage facility in the absence of operational costs. To see this, imagine that all options are exercised against us: we then simply settle the loss incurred from our counterparties exercising and instantly offset this loss by taking reversed positions in the underlying forward spreads – thus locking in the cashflows given by the spreads currently prevailing in the market.

For the purposes of the following example, we employ the

5. A long calendar spread call on two forwards means that the buyer assumes a long position in the shorter dated month and a short position in the longer dated month. This arrangement represents the positions we would assume under the rolling intrinsic strategy.

T4. Calendar spread option premiums associated with all unordered pairs of forward prices (pence/therm) *Source: Lacima Group*

		Withdrawal										
		May 07	Jun 07	Jul 07	Aug 07	Sep 07	Oct 07	Nov 07	Dec 07	Jan 08	Feb 08	Mar 08
Injection	Apr 07	0.18	0.46	0.01	0.18	-	8.28	25.10	31.28	34.69	32.72	26.06
	May 07	-	0.57	0.18	0.38	0.01	8.18	25.00	31.18	34.59	32.62	25.96
	Jun 07	-	-	0.06	0.23	0.01	7.84	24.66	30.84	34.25	32.28	25.62
	Jul 07	-	-	-	0.64	0.04	8.69	25.51	31.69	35.10	33.13	26.47
	Aug 07	-	-	-	-	0.01	8.18	25.00	31.18	34.59	32.62	25.96
	Sep 07	-	-	-	-	-	9.90	26.72	32.90	36.31	34.34	27.68
	Oct 07	-	-	-	-	-	-	16.82	23.00	26.41	24.44	17.78
	Nov 07	-	-	-	-	-	-	-	6.18	9.59	7.62	1.39
	Dec 07	-	-	-	-	-	-	-	-	3.46	1.75	0.01
	Jan 08	-	-	-	-	-	-	-	-	-	0.17	-
	Feb 08	-	-	-	-	-	-	-	-	-	-	-

T5. Optimal monthly injections and withdrawals under the basket of spreads strategy (MMBtu) *Source: Lacima Group*

		Withdrawal												
		May 07	Jun 07	Jul 07	Aug 07	Sep 07	Oct 07	Nov 07	Dec 07	Jan 08	Feb 08	Mar 08	ΣInjection	Constraint
Injection	Apr 07	-	-	-	-	-	-	245,910	-	-	-	-	245,910	245,910
	May 07	-	245,910	-	-	-	-	-	-	-	-	-	245,910	245,910
	Jun 07	-	-	-	229,550	-	-	-	16,360	-	-	-	245,910	245,910
	Jul 07	-	-	-	-	-	-	-	33	-	245,877	-	245,910	245,910
	Aug 07	-	-	-	-	-	-	-	-	245,893	17	-	245,910	245,910
	Sep 07	-	-	-	-	-	-	-	-	245,910	-	-	245,910	245,910
	Oct 07	-	-	-	-	-	-	-	-	-	-	-	-	-
	Nov 07	-	-	-	-	-	-	-	245,910	-	-	-	245,910	245,910
	Dec 07	-	-	-	-	-	-	-	-	-	245,910	-	245,910	245,910
	Jan 08	-	-	-	-	-	-	-	-	-	-	-	-	-
	Feb 08	-	-	-	-	-	-	-	-	-	-	245,910	245,910	245,910
	ΣWithdrawal	-	245,910	-	229,550	-	-	245,910	262,303	491,803	491,803	245,910		
	Constraint	491,803	491,803	491,803	491,803	491,803	491,803	491,803	491,803	491,803	491,803	491,803		

same facility characteristics and notation as above, but our optimisation problem can now be stated as:

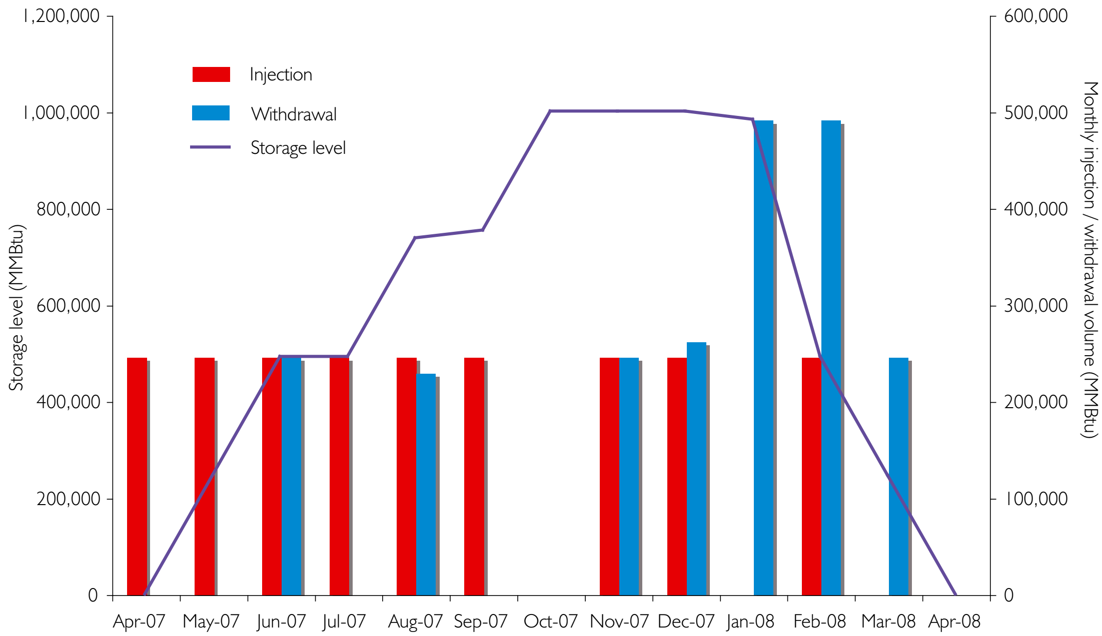
$$\max \sum_i \sum_j v_{ij} C_{ij}\left(F_i;\ F_j;\ T_i;\ T_j;\tau\,;\Theta\right)$$

subject to the constraints specified above and where C_{ij} is the price of the calendar spread call option as a function of the two forwards F_i and F_j, their terms to maturity T_i and T_j, the term to expiry of the option τ, and Θ, which is a vector of parameters that depends on the respective pricing model specification. In the case of the underlying stochastic process being a mean-reversion one as proposed by Clewlow & Strickland (2000), Θ would be composed of the volatilities associated with F_i and F_j, the mean reversion rate of the process, and the correlation between the log forward returns. Table 4 shows option premiums calculated on March 31, 2007 for the NBP forwards used in our earlier example where the parameters of the mean-reverting model chosen to run this example are calibrated to historical data during the five-year period preceding the valuation date.

Each combination in table 4 represents the price of a calendar spread option where we pay the price of the underlying forward given by the corresponding column header and receive the price of the underlying forward given by the corresponding row header upon exercise. Taking the Apr–May pair as an example, we have thus sold an option to pay May and to receive April in the case of the option being exercised. From table 4 we can also see that the highest premium

T6. Position adjusted revenues under the basket of spreads strategy (£) *Source: Lacima Group*

		Withdrawal											
		May 07	Jun 07	Jul 07	Aug 07	Sep 07	Oct 07	Nov 07	Dec 07	Jan 08	Feb 08	Mar 08	ΣInjection
Injection	Apr 07	-	-	-	-	-	-	617,222	-	-	-	-	617,222
	May 07	-	14,121	-	-	-	-	-	-	-	-	-	14,121
	Jun 07	-	-	-	5,349	-	-	-	50,453	-	-	-	55,803
	Jul 07	-	-	-	-	-	-	-	106	-	814,573	-	814,679
	Aug 07	-	-	-	-	-	-	-	-	850,528	55	-	850,582
	Sep 07	-	-	-	-	-	-	-	-	892,881	-	-	892,881
	Oct 07	-	-	-	-	-	-	-	-	-	-	-	-
	Nov 07	-	-	-	-	-	-	-	151,983	-	-	-	151,983
	Dec 07	-	-	-	-	-	-	-	-	-	42,920	-	42,920
	Jan 08	-	-	-	-	-	-	-	-	-	-	-	-
	Feb 08	-	-	-	-	-	-	-	-	-	-	6	6
	ΣWithdrawal	-	14,121	-	5,349	-	-	617,222	202,542	1,743,409	857,547	6	



F3. Potential storage level and monthly injection and withdrawal profile *Source: Lacima Group*

can be received by selling the Sep 07–Jan 08 spread option amounting to 36.31 pence, which is slightly higher than the position in the underlying forward spread.

Table 5 gives us the resulting optimal injection and withdrawal rates that are obtained by maximising the portfolio value which is given by the sum of products of the individual optimal volumes and their corresponding spread option premiums.

In table 5, the sum of each row represents the injections for the month corresponding to the row header, whereas the sum over each column represents the withdrawals for the month corresponding to the column header. For example, we have total injections of 245,910 MMBtu for Apr 07 and total withdrawals of 491,803 MMBtu for Nov 07. Assuming the Apr 07–Nov 07 spread option was exercised this would imply that we have to financially settle the cashflow arising from the Apr 07–Nov 07 forwards difference multiplied by 245,910 MMBtu. To offset this loss we immediately go long the Apr 07 forward and short the Nov 07 forward and have thus secured the option premium. The resulting revenues from each portfolio position are displayed in table 6 and yield a storage value of £3,440,196.

A profile of the monthly injections and withdrawals and the resulting storage level over the term of the contract is displayed in figure 3.

Comparing the results for the two models described above we can see that the basket of spread options strategy leads to an increase in value of around 8%. We note, however, that the extra time value created depends heavily on the parameterisation of our pricing model – particularly our estimates of the future volatilities and correlations.

Note that for both the intrinsic and the basket of spread options strategies, from a valuation perspective these are static strategies – the volume positions are fixed at the start of the period based on the initial forward curve and, for the spread option strategy, a view of the volatility and correlations in the market. In practice the spread option value is sometimes extracted via a set of aggregate delta hedges, which change dynamically through time dependent on the evolution of the forward curve. This strategy is therefore analogous to the rolling intrinsic strategy, which extends the simple intrinsic strategy described above to allow for adjustments to the volume positions as the forward curve evolves through time. This valuation methodology will be the subject of the next article in our Masterclass series. ER

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